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A Coding Approach to Reduce Peak-to-Average Power Ratio in Coded OFDM

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ABSTRACT

High peak-to-average power ratio has been major issue in the implementation of orthogonal frequency division multiplexing systems in low cost application devices. In this paper, we propose an peak-to-average-ratio reduction technique which uses channel coding especially expurgated codes. This method combines the error correction capability of the channel encoder and it achieves PAPR reduction in the OFDM system. Our technique is easily implemented in terms of encoding as well as decoding. We shall discuss the trade-o?s involved in our method.

INSPEC Classification : B6100, B6250.

1. INTRODUCTION

The wireless industry is undergoing a major evolution from narrow-band, circuit switched legacy sys-tems to broadband, IP-centric platforms. A common theme in this broadband evolution is the use of orthogonal frequency division multiplexing(OFDM) modems and open network architectures. OFDM has become one of the most exciting developments in the area of modern broadband wireless networks. Although the notion of multicarrier transmission or multiplexing (e.g., frequency-division multiplexing - FDM) can be dated back to 1950s, its true potential is becoming obvious in WiMax and IEEE 802.16. One of the major drawbacks in implementing OFDM is its high peak-to-average power ratio (PAPR). High PAPR suggests use of expensive ampli?ers with a large dynamic performance range. If the peak transmit power is limited by either regulatory or application constraints, the e?ect is to reduce the average transmission power, which reduces the transmission range. This is not a feasible option for low-cost applications. Due to high PAPR, the transmit power ampli?er must operate in a region where the power conversion is ine?cient. In the low-cost application, the potential bene?ts of the OFDM are overshadowed by the drawbacks of high PAPR.

OFDM which applies channel coding is called Coded OFDM (COFDM) [Zou 95]. Coding is becoming an essential part of OFDM employed modulation schemes such as WLAN, digital audio broadcast (DAB), digital video broadbast (DVB), 802.16 and WiMax.

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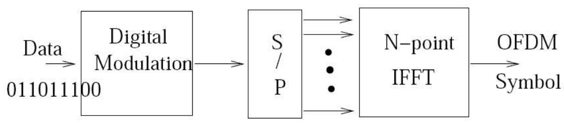
It would make perfect sense to use coding method to control the high PAPR in OFDM. Thus, we can utilize coding for twin purpose; to correct errors as well as to control PAPR. Coding has been applied to alleviate the problem of PAPR in OFDM (Barton 94, Davis 99, Jones 96, Omar 04). But these coding approaches su?er from the need to exhaustively search for effective codes, store large look-up tables or do large computations. For example, Davis' paper discusses extraction of codes whose PAPR characteristics are very favorable but require large number of computations for encoding as well as decoding.

A coding approach is proposed which enjoys the twin bene?ts of the error correction as well as reducing PAPR value in COFDM. The method is easy to implement using shift registers. The method is based on grouping a given code C into smaller groups. This particular grouping generates multiple codewords for a given message block. Each of these codewords in a group has a di?erent PAPR value for the OFDM symbol. The codeword is chosen according to the preset threshold PAPR value. In

Figure 1.
Block diagram of OFDM system



this article, an adaptive or iterative codeword developing method is also suggested based on expurgated codes to control PAPR in COFDM. The rationale to use adaptive method is to allow termination of search for the best codeword within a group. Search for a codeword is terminated, when PAPR of an OFDM symbol is found to be below a present threshold value.

A simple implementation method is proposed using classical coding theory. Small groups within the code set by using certain encoding procedure. Our approach does not need any side information to be sent along the data or power increase at the transmitter. The main advantage of our approach is that the decoding is very simple. The rest of the article is organized as follows:

2. PAPR IN OFDM

The basic block diagram of the OFDM system is shown in Figure 1. In this paper, we consider an OFDM modulation with a total of n-subcarriers. Let a message block with k symbols in it be represented as $\mathbf{m} = (m_0, m_1, ..., m_{k-1})$ and encoded data is represented as $\mathbf{c} = (c_0, c_1, ..., c_{n-1})$. Each symbol in c is an element of GF (q). Each symbol in c is then mapped into a complex value using some digital modulation schemes such as BPSK, QPSK, M-QAM to get the complex vector $\mathbf{X} = (X_0, X_1, ..., X_{n-1})$. As shown in the block diagram, the vector X undergoes Inverse Fast Fourier Transform (IFFT) to acquire the transmitted COFDM signal in time domain.

$$\mathbf{x}(t) = \sum_{i=0}^{n-1} X_i e^{j2\pi i \Delta f t} \tag{1}$$

where $j = \sqrt{-1}$, $\Delta f = f_{i+1} - f_i$ is a integer multiple of time period of OFDM symbol T. This is chosen in order to maintain the orthogonality of subcarriers frequencies. The equivalent l sampled representation of the time signal $x(t)$ is given by

$$\mathbf{x}(l) = \sum_{i=0}^{nL-1} X_i e^{j2\pi i l} \tag{2}$$

where each symbol of $x(l)$ is represented as x_l. Here L is the oversampling factor usually greater than or equal to 4.

The discrete time PAPR of an OFDM signal of (2) can be de?ned as following ratio:

$$P(x) = \frac{\max |x_l|^2}{E\{|x_l|^2\}} \tag{3}$$

where $E\{.\}$ is the expectation operator. The cumulative distribution function (CDF) of the PAPR is used as performance measure for PAPR reduction techniques. In the literature, the complementary CDF (CCDF) is commonly used instead of the CDF itself. The CCDF of the PAPR denotes the probability that the PAPR of a data block exceeds a given threshold.

$$P_{papr} = P_r(P > P_t) \tag{4}$$

A simple approximate expression is derived for the CCDF of the PAPR of an OFDM signal with Nyquist rate sampling.

$$P_r(P > P_t) = 1 - [1 - \exp(-P_t)]^n$$

where n is the number of subcarriers.

3. BASICS OF CYCLIC CODES

Our coding approach is based on the expurgated codes of the linear cyclic codes. Expurgated codes are generated by shortening the message length block, which in turn discards some codewords. In this paper, we show the complete process of encoding and decoding for our coding approach. Later, example A is used to illustrate the working of our approach.

Coding theory is a well established field of mathematics and computer science and has been ex- tensively used in communications engineering since Shannon showed that improvement in BER can be achieved by use of channel coding. We use coding theory to suggest a simple but effective approach to tcontrol PAPR in OFDM systems.

Consider a (n, k, t) linear cyclic code-C, where n is the length of the codeword, k is the number of message symbols, and t is the number of correctable errors. The elements of $C \varepsilon Z_q$. The code rate r_c is defined by the dimensionless ratio k/n. Encoding is the process of mapping k-symbols to n-symbols, which is usually performed using the generator polynomial g(D). The generator polynomial, g(D), is an irreducible polynomial factor of $1 + D^n$. If the degree of g(D) is j such that $n = 2^j - 1$ then it is called the primitive polynomial. Another polynomial that is associated with any cyclic code is the parity-check polynomial, h(D). The relation between the generator and the parity-check matrix of any linear code is given by:

$$h(D) \cdot g(D) = 1 + Dn \tag{6}$$

We propose to use expurgated code, C_e of form $(n, k + \alpha)$, where a is the number of symbols such that $(\alpha < k)$. The expurgated codes are generated by discarding some of the codewords from original codes C, thus C_ed C.

One common way of obtaining $g_e(D)$ is as follows:

$$g_e(D) = x(D).g(D) \tag{7}$$

where $x(D)$ is a factor of $1 + D^n$ of degree α. The mapping of code rate k/n is accomplished by using effcient cyclic encoding process.
Therefore, if $g_e(D) = x(D).g(D)$, when $x(D)$ is a factor of $1 + D^n$. Then parity-check polynomial $h_e(D)$ can be obtained as follows:

$$h_e(D) = \frac{1 + D^n}{g_e(D)} \quad (8)$$

4. OUR CODING APPROACH

Our coding method is based on the expurgated codes, which are generated by shortening the message length block. We propose a structured grouping of codewords for the block of k symbols that can be easily decoded at the receiver. Each group has one-to-one correspondence for each k-symbol. The codeword with least PAPR is selected among each group. No side information needs to be sent along with data for decoding purpose. Consider a block **m** = $(m_0, m_1, ..., m_{k-1})$ consisting of k symbols belonging to Z_q. A new block **m'** = $(m_k, ..., m_{k+\alpha-1}, m_0, m_1, ..., m_{k-1})$ is formed, by affixing α-symbols to the preexisting block **m**. The α value could be greater than k, but should be less than n-k. The polynomial representation of **m** and **m'** are as follows [Simon 88]:

$$\mathbf{m}(D) = m_0 + m_1 D + ... + m_{k-1} D^{k-1} \quad (9)$$

$$\mathbf{m'}(D) = m_k + ... + m_{k+\alpha-1} D^{\alpha-1} + ... + m_{k-1} D^{k+\alpha-1} \quad (10)$$

The **m'** has a total of $k + \alpha$ symbols. Depending upon the coeffcients of the first α terms in (8), we have q^α combinations in which we can augment **m** to generate **m'**. Let the different representations of **m'** be labeled as $\mathbf{m'}_1, \mathbf{m'}_2, ..., \mathbf{m'}_{q^\alpha}$. Using the linear cyclic code generator polynomial g(D), each of the $\mathbf{m'}_i$ for $i = 1, ..., q^\alpha$ is encoded to form (n, $k + \alpha$, t) codeword represented as $c_i(D)$ for $i = 1, ..., q^\alpha$. The general form of the systematic codeword using the generator polynomial g(D) is given by

$$c(D) = b(D) + D^{n-k-\alpha}\mathbf{m'}(D) \quad (11)$$

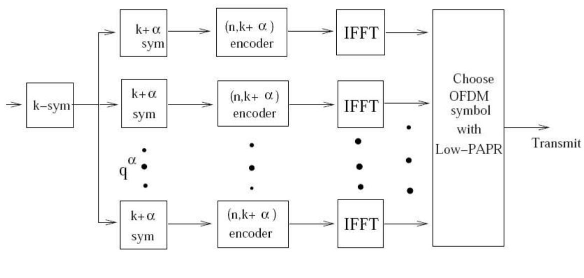
where $b(D) = D^{n-k-\alpha}\mathbf{m'}(D) - a(D)g(D)$, i.e., the remainder when $D^{n-k-\alpha}\mathbf{m'}(D)$ is divided by g(D). Note that all these operations are done over $GF(q)$. This linear cyclic code has a correction capability of up to t symbols. The initial code rate is k/n.

The q^α representations are used to form OFDM symbols using the Inverse Fast Fourier Transform (IFFT) on separate branches as shown in Figure 2. Let the PAPR value of each of the different OFDM symbol be represented as P_i for $i = 1, ..., q^\alpha$. The PAPR value of each representation is calculated, and the representation with least PAPR value is selected.

$$P = min\{P_i\} \qquad i = 1, ..., q^\alpha$$

The selected representation is transmitted as the OFDM signal. The remaining q^α-1 representations are ignored or discarded.

Figure 2.
Block diagram of Encoder



At the decoder, the received OFDM signal is de-multiplexed using Fast Fourier Transform (FFT) followed by decoding. Decoding consists of two steps: error correction and message recovery. Let the received word be represented as **r**(*D)* given by

$$r(D) = c(D) + e(D) \tag{13}$$

where e(*D*) is the error pattern polynomial. This error pattern can be identi?ed using syndrome polynomial s(*D*) calculated as follows:

$$s(D) = r(D) - a(D)g(D) \tag{14}$$

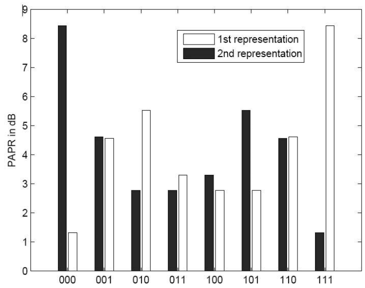
meaning s(*D*) is the remainder polynomial when r(*D*) is divided by g(*D*). There is a one-to-one correspondence between s(*D*) and the error pattern. Once the received word is corrected of errors, the next step is to recover our initial *k*-symbols message. Let $c_e(D)$ be the corrected codeword polynomial. Then, the estimate of **m'**(D) is recovered by extracting the coeffcients of *n*- *k*, *n*- *k* + 1, ..., *n*-1 degree terms. This can be done conveniently because of the systematic structure of the codeword.

4.1 Example A

We shall demonstrate our approach using a simple example. Consider a 3-symbol message block, where $q = 2$, say m = 110. We affix one symbol ($\alpha = 1$) to this message to get a 4-symbol block; there are $2^{\alpha} = 2$ different combinations to append. For this example, the two new blocks are $\mathbf{m'}_1 = 0110$ and $\mathbf{m'}_2 = 1110$. Using the (7, 4, 1) linear cyclic code, we shall encode $\mathbf{m'}_1$ and $\mathbf{m'}_2$ into $c_1$1and c_2 respectively using a generator polynomial such as $g(D) = 1 + D^2 + D^3$. Thus, the parity-check polynomial will be theproduct, $h(D) = (1 + D)(1 + D + D^3)$. The codewords corresponding to $\mathbf{m'}_1$ and $\mathbf{m'}_2$ are $c_1 = 0010110$ and $c_2 = 1001110$, respectively. These two representations are used to develop the OFDM symbols, and PAPR of each representation is calculated. Figure 3 shows the PAPR values of both representations for all three bit words. Let P_1 and P_2 are PAPR values of these two representations; the representation with minimum PAPR is choosen for transmission.

At the decoder, the received word is decoded using the syndrome polynomial to retrieve the original 3-bit sequence. This method is simple to implement in terms of encoding and decoding. Figure 3 shows the amount of reduction achieved with our approach for $\alpha = 1$ in the above example. Clearly, a maximum reduction of 5*dB* is seen for this case.

Figure 3.
Amount of PAPR reduction for $\alpha = 1$



5. BER TRADE-OF ANALYSIS

In this section, we shall analyze the trade-offs involved with probability of error. Even though the encoding is done using code rate $r_c = (k + \alpha)/n$, the initial code rate as noted before is *k*/*n*. We know that when the code rate is increased, the probability of error also increases. The increment in code ratercis given by:

$$\Delta r_c = \frac{k+\alpha}{n} - \frac{k}{n} = \frac{\alpha}{n} \qquad (15)$$

We will show that the amount of increment in the probability of error is negligible, when code rate is increased by α/n. It is well documented that the error probability of the binary channel for BPSK signals is given by [Proakis, p.447]:

$$p = Q\left(\sqrt{\frac{2r_c E_b}{N_0}}\right) \qquad (16)$$

where E_b/N_0 is signal-to-noise ratio, and

$$\mathbf{Q}(x) = \frac{1}{2}\, erfc\left(\frac{x}{2}\right).$$

The probability of message-error is upper bound by [Proakis, p.456]:

$$p_e < (M-1)[4p(1-p)]^{d_{min}/2}$$

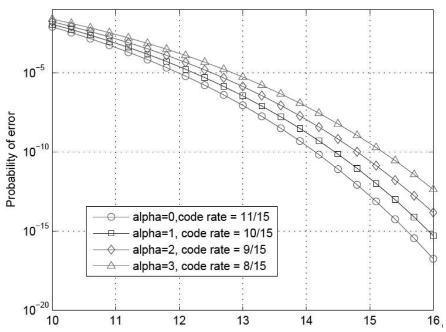
where M is the number of codewords (2^k), and dminis the minimum Hamming distance of the code.

For a given E_b/N_0, the increment in probability of error for α = 0, 1, 2, 3 and n = 15, k + a = 11 is shown in Figure 8. The increment for low SNR is nearly zero, whereas for high SNR, the increment is very small, almost negligible. Thus, we believe that this trade-off is more advantageous compared to the amount of reduction achieved in PAPR.

6. Adaptive Coding PAPR reduction technique

Consider a linear code (n, K), such that every K-symbol message word is mapped onto a n-symbol codeword. Thus, the coding rate is given by $r_c = K/n$. Elements of our linear code belong to alphabet of size q. Let us consider k message symbols and append a symbols at the begining such that we have $k + \alpha = K$ symbols. Depending upon the values of α symbols, we can exhaustively form q^α new message sequences of length K. Thus, for any message word of length k, we could generate q^α new message sequences of length K.

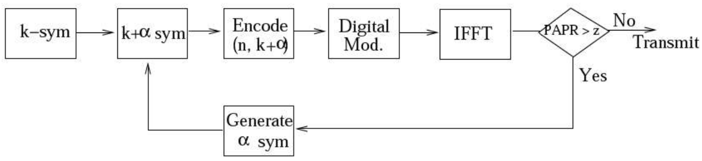
Figure 4.
Probability of error for n = 15, k + α = 11



The new message sequences are encoded using linear code (n, K), generating different codewords for each message sequence. The different codewords thus generated have different PAPR characteristics, presenting an opportunity to choose a codeword with PAPR value of choice. We shall elaborate our method using detailed notations.

Consider a block **m** = (m_0, m_1, ..., m_{k-1}) consisting of k symbols belonging to Z_q. A new block **m'**= (mk, ..., mk+α-1, m0, m1, ..., mk-1) is formed, by affxing α symbols to the preexisting block **m**. The α value should be less than k. Each one of the representations of the message **m'** is encoded using linear code (n, K) in systematic or non-systematic form. We begin by selecting one such representation from **m'** to encode the data and find PAPR of that OFDM symbol. If the PAPR value of the OFDM symbol is less than the preset threshold z, then the symbol is transmitted. If the PAPR value of the OFDM symbols is greater than the preset threshold z, then a different representation is selected from **m'** and the PAPR value is evaluated again. This process is repeated until a representation is found that satisfies the condition of the PAPR value.

Figure 5.
Block diagram of adaptive PAPR reduction



Adaptive coding Algorithm for PAPR reduction

1. Begin with k-symbol message sequence
2. Append α symbols at the end of message
3. Encode using (n, $k + \alpha$)
4. Evaluate PAPR of OFDM symbol
5. if PAPR > z (threshold level)
6. then goto step 2
7. else
8. then transmit OFDM signal
9. Decode using (n, $k + \alpha$) to get $k + \alpha$ symbols
10. Extract k symbols to get original message sequence

6.1 Probability analysis of our approach

We propose to create multiple copies of a message sequence by appending α symbols at the end. De-pending upon the values of α symbols, we can form many message sequences for each original message **m**. Using encoding each representation of **m** in the form of **m'**is mapped onto one codeword of length n. Consequently, each original message block **m** has many different valid codewords. In other words, any one of the q^{α} diff.erent codewords can be used to decode the original message **m**.

We shall show mathematically how our approach of grouping codewords increases the probability of reducing the PAPR of OFDM symbols. Using the concept of combined experiments from probability, let us deffine two experiments ψ_1 and ψ_2. Experiment $\psi_1$is the selection of a message sequence **m** of length k. The sample space 1_1 of this experiment is all the possible message sequences of length k. Assume that all the message sequences are equiprobable. The probability of selecting message **m** is given by :

$$P\,(\mathrm{m}) = \frac{1}{q^{k}}.$$

Experiment ψ_2 is the selection of an *n*-tuple codeword, c. The sample space 1 2 of this experiment is the set of all valid codewords. Since there is a one-to-one correspondence between codeword and message sequence, we have probability of selecting a valid codeword under normal coding given by:

$$P\,(c) = \frac{q^k}{q^n}. \quad (19)$$

Considering experiment ψ as combined experiment of ψ_1 and ψ_2 Thus, $\psi_1 = (\psi_1, \psi_2)$. We haveprobability of selecting message **m** and selecting a valid codeword c as follows [Papoulis, p.49]:

$$P(m, c) = P\,(m).P\,(c) = \frac{1}{q^n}. \quad (20)$$

Under expurgated coding method, since there are multiple version of message sequence for a single message **m** denoted as **m'**. The probability of selecting message **m** becomes

$$P(\mathbf{m'}) = \frac{q^\alpha}{q^k}. \quad (21)$$

Since each message has a unique codeword, the probability of selecting a valid codeword is given by (19). The combined probability of selecting a codeword for a given message sequence is given by

$$P\,(\mathbf{m'}, c) = P\,(\mathbf{m'}).P\,(c) = \frac{q^\alpha}{q^n}. \quad (22)$$

When α is non-zero, (19) and (22) suggests that multiple codewords (i.e. up to q^α) can be decoded into same message sequence of length *k*. This introduced redundancy or grouping of codewords that can be decoded into one single message sequence is the key for reduction of PAPR by this technique. This redundancy can be thought of as a type of gain given by a ratio of (22) and (20) as follows:

$$G = \frac{P\,(\mathbf{m'}, c)}{P\,(\mathbf{m'}, c)} = q^\alpha \quad (23)$$

Let *z* be a random variable representing the PAPR value of an OFDM symbol. For one OFDM symbol with *n* subcarriers, the complex baseband signal can be written as (1). From the central limit theorem, it follows that for large values of *n*, the real and imaginary values of $S_u(t)$ becomes Gaussian distributed, each with a mean of zero and a variance of 1/2 [Prasad, p.120]. The amplitude of the OFDM signal follows a Rayleigh distribution and its power distribution follows the central chi-square distribution with two degress of freedom and zero mean, whose cumulative distribution is:

$$F_z(z) = 1 - e^{-z}. \quad (24)$$

The probability that the PAPR for an OFDM symbol is below threshold level *z* is thus approximately equal to the probability that all *n* samples in time are below threshold. Assuming that the samples are mutually uncorrelated (even though this assumption does not hold as α increases), the cumulative distribution function can be written as [Prasad, p.121]:

$$P(z \# z) = (1 - e^{-z})^n \quad (25)$$

Consequently, the probability that *z* exceeds a certain threshold is given by:

$$P\,(z > z) = 1 - (1 - e^{-z})^n \quad (26)$$

Now, assuming that our redundancy or gain *G* indicates statistically independent OFDM symbols rep-resenting the same information. The probability that z exceeds the threshold *z* in this situation is given by:

$$P(z > z) = [1 - (1 - e^{-z})^{n}]^{G} \quad (27)$$

Figure 6.
CCDF curves according to eqn 26 (solid lines with markers) and 27 (dotted lines with markers) for n = 16, 32, 64, 128, 256 from left to right

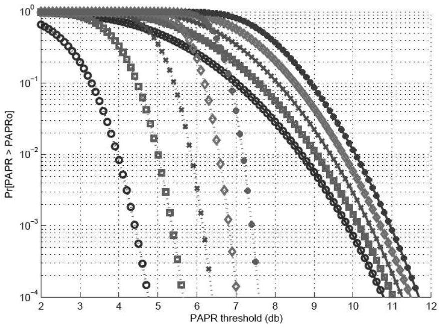


Fig. 6 shows the potential of our expurgated coding technique for $\alpha = 4$ and $q= 2$ using (26) and (27). The solid line plots and dotted line plots with same markers form a pair, showing the CCDF curves with no PAPR technique and the CCDF curves with our expurgated coding technique respectively. The values of *n* are 16, 32, 64, 128, 256 from left to right. The depicted is the maximum possible reduction achievable under assumptions made in the derivation of equations plotted. We shall compare the simulation results to these predictions in section 9.

7. AVERAGE NUMBER OF ITERATIONS REQUIRED

In this section, we develop the expression for the expected value of the number of iterations required to achieve a PAPR lower than the threshold. In the adaptive approach, the iterations are stopped as soon as PAPR value reaches below threshold. For notational convenience, we rewrite (26) as follows:

$$f = 1 - (1 - e^{-z})^{n}. \quad (28)$$

Let $p^{(i)}(z > z)$ be the probability that the PAPR value is greater than the threshold with *i* iterations given in terms of new notation as

$$p^{(i)}(z > z) = f^{i}, \qquad (1 \# i \# I), \quad (29)$$

where *I* be the maximum number of iterations specified in the adaptive approach. Recall that each iteration is related the generation of a symbols to form new message block **m'**. Let i^{th} be the iteration on which the adaptive procedure stops, then it implies that for the first *i* - 1 iterations the PAPR was greater than threshold and on the i^{th} iteration the PAPR was less than threshold. Let i_i be the event that the iterations will stop on i^{th} iteration. Therefore,

$$P(i_1) = 1 - f,$$
$$P(i_2) = f(1 - f),$$
$$P(i_3) = f^{2}(1 - f),$$

and similarly for

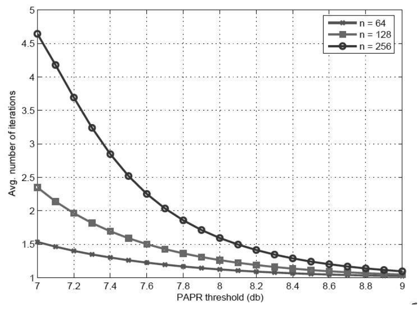
$$P(i_i) = f^{i-1}(1 - f),$$

The expected value, $E\{I\}$, of the number of iterations required to achieve PAPR less than threshold is given by

$$E\{I\} = \sum_{i=1}^{I} iP(i_i)$$

$$E\{I\} = \sum_{i=1}^{I} i f^{i-1}(1 - f). \qquad (30)$$

Figure 7.
Theoretical average number of iterations needed for a given threshold for n = 64, 128 and 256



In Fig. 7, the average number of iterations needed for a given threshold value is given for various values of n. As the threshold decreases the number of iterations increases and so is the case for number of subcarriers.

8. TRADE-OFF ANALYSIS

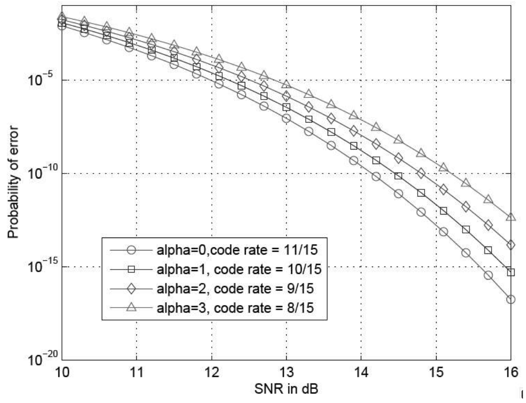
In this section, we shall analyze the trade-offs involved with probability of error. Even though the encoding is done using code rate $r_c = (k + \alpha)/n$, the initial code rate as noted before is k/n. We know that when the code rate is increased, the probability of error also increases. The increment in the code rate r_c is given by $\Delta r_c = \frac{k+a}{n} - \frac{k}{n} = \frac{a}{n}$. We will show that the amount of increment in the probability of error is negligible, when code rate is increased by a/n. It is well documented that the error probability of the binary channel for BPSK signals is given by [Proakis, p.447]:

$$p = Q\left(\sqrt{\frac{2r_c E_b}{N_0}}\right)$$

where E_b/N_0 is signal-to-noise ratio, and

$$Q(x) = \frac{1}{2}\, erfc\left(\frac{x}{2}\right)$$

Figure 8.
Probability of error for n = 15, k + α = 11



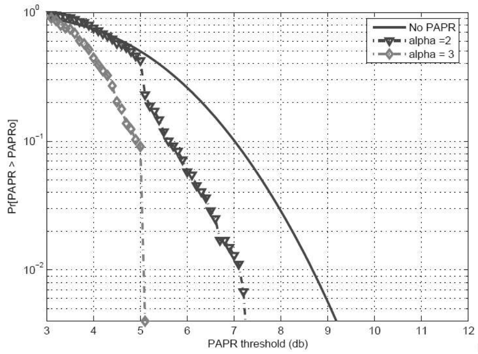
The probability of message-error is upper bound by [Proakis, p.456]:

$$p_e = (M - 1)[4p(1 - p)]^{d_{min}/2} \quad (32)$$

where M is the number of codewords (2^k), and dminis the minimum Hamming distance of the code. For a given E_b/N_0, the increment in probability of error for α = 0, 1, 2, 3 and n = 15, k + α = 11 is shown in Fig. 8. The increment for low SNR is nearly zero, whereas for high SNR, the increment is very small, almost negligible. Thus, we believe that this trade-off is acceptable compared to the amount of reduction achieved in PAPR.

9. SIMULATION RESULTS

Figure 9.
CCDF of PAPR for n = 16, a = 2,3



We developed simulations to quantify our method's success in terms of CCDF plots. The number of subcarriers used are both small and large in number, namely n = 16 and 64. Binary codes are used in these preliminary simulations, with BPSK modulation. As we know, the closer the CCDF plot is to the y-axis the more effective the PAPR reduction technique is. In Figure 9 and Figure 10, as a increases the plot gets closer to the y-axis indicating the reduction in PAPR. Figure 9 shows that for α = 3, there is less than 0.1%

chance that the PAPR will be greater than 5*d*B. Similarly for n = 64, as a increases the amount of PAPR reduction increases gradually, as is shown in Figure 10.

Figure 10.
CCDF of PAPR for n = 64, α = 2,4,6

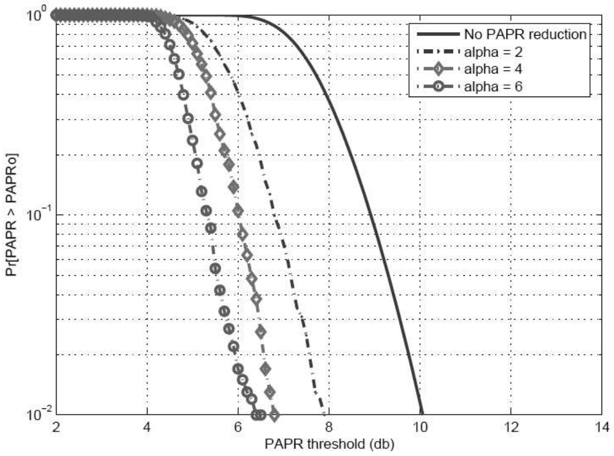
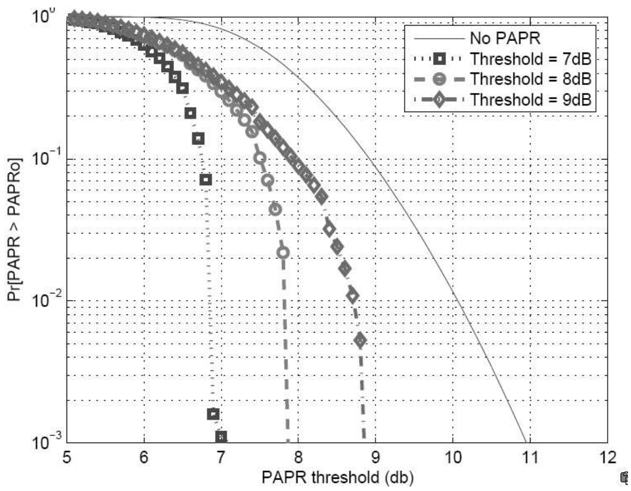
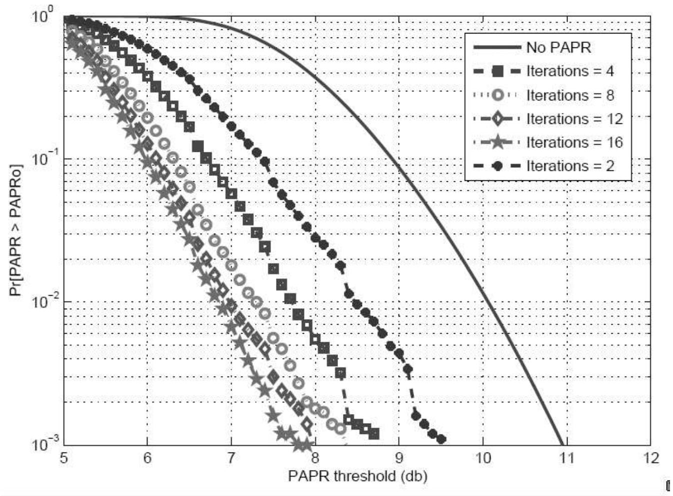


Figure 11.
CCDF showing reduction of PAPR when threshold at given values



An OFDM system with various values for number of subcarriers with BPSK modulation is simulated to obtain the following results. Fig. 11 depicts the simulated performance of our expurgated coding method with n = 64, k = 47 and α = 5 for different values of threshold level namely 9dB, 8dB and 7dB. Clearly, it shows that only those OFDM symbols were selected which satisfied the PAPR threshold condition. In our adaptive implementation, the maximum number of iterations, of q^{α} are not used. In fact, iterations stop as soon as the threshold condition is met. For completeness, we plotted in Fig. 12 the reduction of PAPR for a given number of iterations such as 2, 4, 8, 12, and 16. As expected, the amount

Figure 12.
CCDF showing reduction of PAPR with given number of iterations



amount of PAPR reduction increases as the number of iterations increased. The simulation results confirm the analytical prediction made by 27.

10. CONCLUSION

In this paper, we described a coding method for reducing PAPR in COFDM. Our method combines both aspects into one, to achieve reduction in PAPR as well as error correction capability. The adaptive approach is adopted in order to reduce hardware for a slight increase in complexity. The reduction in PAPR is estimated based on some general assumptions and the simulation results con?rm the analysis. This approach can be implemented with large number of subcarriers, and any modulation. The trade-off involved with the probability of error increment is negligible compared to the amount of PAPR reduced.

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